

Oberseminar Algebraic Geometry WS 07/08

Deligne's theory of mixed Hodge structures

The classical Hodge theorem states that the cohomology of a compact Kähler manifold can be decomposed as

$$H^n(X, \mathbf{C}) = \bigoplus_{p+q=n} H^{p,q} \qquad \overline{H^{p,q}} = H^{q,p}$$

where $H^{p,q}$ can be characterized as hypercohomology of holomorphic differential forms $H^p(X, \Omega^q)$. In particular, this applies to smooth projective varieties over $\mathbf{C}$. A vector space with such a structure is called *pure Hodge structure of weight n* . In his thesis, Deligne has shown how this can be generalized to non-projective (i.e., non-compact) smooth varieties and even to singular ones. He introduces the notion of mixed Hodge structures and constructs mixed Hodge structures on the cohomology of any variety over $\mathbf{C}$.

Our aim this semester is to read Deligne's thesis carefully, taking most of the analytic work behind it for granted. All participants are welcome. Some background in algebraic geometry and homological algebra is necessary.

Room: Felix-Klein-Hörsaal, Johannissgasse 26.

Time: Tue 15-17

First meeting: October 9, 2007.

All references refer to P. Deligne, *Théorie de Hodge II*, Publ. IHES 40 (71) p. 5–57. The last talks use P. Deligne, *Théorie de Hodge III*, Publ. IHES 44 (74), 5–77.

0. (Introduction) Give definition 2.3.1 (mixed Hodge structures), state Theorem 2.3.5 and give a short discussion of why this is difficult. State the main result of Section 3.2: existence of a mixed Hodge structure on cohomology of smooth algebraic varieties. Give a survey of the construction, in particular in the smooth projective case.
1. (Filtered objects) 1.1.-1.2.4
2. (Opposed filtrations) rest of 1.2

3. (Lemma of two filtrations) Introduce the notion of spectral sequence, degeneration of spectral sequences etc. (any book on homological algebra) and in particular the spectral sequence attached to a filtered complex. Follow Deligne for normalizations. Present in particular 1.3.1-1.3.5
4. (More Spectral sequences) Rest of 1.3, quickly review hypercohomology 1.4.1-1.4.5.
5. (Hypercohomology spectral sequence) Present the properties of hypercohomology which we need later rest of 1.4.
6. (Hodge structures) Define the category of mixed Hodge structures and its properties 2.3, also discuss pure Hodge structures 2.1
7. (Hodge structure of a smooth projective variety) Present 2.2 and give a survey on the analysis behind it, i.e. the Theorem of Hodge.
8. (Differentials with logarithmic singularities) Introduce differentials with logarithmic singularities and their properties 3.1.
9. (Regular singular differential equations) Give a survey on Lecture Notes 163, in particular the proofs of the results used in the previous talk.
10. (The Hodge structure of a smooth variety) Define the mixed Hodge structure for a smooth variety, 3.2 up to 3.2.12
11. (Some Properties) 3.2.13 - 4.1
12. (Cohomology of simplicial schemes) III. 5.1-5.2
13. (Cohomological descent and hypercovers) 5.3 and 6.3. Don't try to present SGA 4 Vbis.
14. (Hodge complexes) 8.1-8.2 Concentrate on getting to the actual definition.
15. Continue previous talk.

For questions:

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